

*IV. Methodus quadrandi genera quædam Curvarum,  
aut ad Curvas Simpliciores reducendi. per A. De  
Moivre R. S. S.*

SIT A Area Curvæ cujus Abscissa  $x$ , & ordinatim Applica-  
ta  $x^m \sqrt{dx-xx}$ . Sit B Area Curvæ cujus Abscissa eadem  
cum priori, sed ordinatim Applicata  $x^{m-1} \sqrt{dx-xx}$ ; ponatur  
 $\sqrt{dx-xx} = y$ . Erit Area A =

$$\begin{aligned}
 & d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} \text{ in } \frac{2m-5}{2m-2} \text{ &c.} = P \\
 & - \frac{1}{m+2} x^{m-1} y^3 = - Q \\
 & - \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} x^{m-2} y^3 = - R \\
 & - \frac{dd}{m} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} x^{m-3} y^3 = - S \\
 & - \frac{d^3}{m-1} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} x^{m-4} y^3 = - T
 \end{aligned}$$

&c.

Ubi notandum 1° quod  $n$  Supponitur numerus integer &  
affirmativus; 2° Quod Quantitas  $d^n B$  in serie per  $P$  designata,  
multipli:carri debet in tot terminos quot sunt unitates  
in  $n$ ; 3° quod tot sequentes series per  $-Q, -R, -S, -T$   
&c. designatae sumi debeant, quot sunt unitates in  $n$ ; quod ut

( 1114 )

Exemplo uno vel altero clarius fiat, dico quod si  $n = 1$ , tunc A

$$= d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3 \text{ & si } n = 2,$$

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-2}{2m+2} - \frac{1}{m+2} x^{m-1} y^3$$

$$- \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} \text{ in } x^{m-2} y^3$$

4o quod si  $y$  ponatur  $= \sqrt{dx-xx}$ , tunc A erit  $= Q - R + S - T$  &c.  $\neq P$ .

### Corollarium.

Si  $m$  ponatur æqualis termino cuivis sequentis Seriei

$$- \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2} \text{ &c.}$$

quadratura Curvæ cuius ordinatim Applicata  $x^m \sqrt{dx-xx}$ , aut  $x^m \sqrt{dx+xx}$  finita evadit & exhibetur per series nostram; quod ut Exemplo illustretur, Inquirenda sit Area Curvæ cuius ordinatim Applicata  $x^{-\frac{1}{2}} \sqrt{dx-xx}$ ; fingatur Curvam hanc comparari cum Curva cuius ordinatim Applicata  $x^{-\frac{1}{2}} \sqrt{dx-xx}$ , quoniam hoc in casu  $n = 1$ , ideo

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3$$

sed  $m = - \frac{1}{2}$ , ergo  $2m+1 = 0$ , ideoq;

$$A = - \frac{1}{m+2} x^{m-1} y^3 = - \frac{2y^3}{3\sqrt{x^3}}$$

Hic Observatu dignum est quod Area sic reperta interdum data quantitate deficit a vera Area, aut eandem data quantitate excedit; quo autem excessus iste aut defectus innoteſcat, ſupponatur Area reperta augeri minuive data quantitate  $q$ , tunc que poſta  $x = 0$ , ſupponatur Area aucta minutave æqualis nihil, ſic in præſenti caſu  $q$  reperietur  $= \frac{2}{3} d \sqrt{d}$ , adeoq;

$$A = \frac{2}{3} d \sqrt{d} - \frac{2 y^3}{3 \sqrt{x^3}}$$

### Corollarium 2<sup>dim.</sup>

Si  $n$  ponatur æqualis termino cuivis ſequentis ſeriei 3, 4, 5, 6, 7, &c. Quadratura Curvæ cuius ordinatim applicata  $x^{-n} \sqrt{dx \cdot xx}$  aut  $x^{-n} \sqrt{dx + xx}$ , finita evadit, & exhibetur per ſeriem noſtram; Inquirenda fit Area Curvæ cuius ordinatim applicata  $x^{-3} \sqrt{dx \cdot xx}$ . finge eam comparari cum Area Circuli, quæ vocetur  $A$ ; erit  $m = 0$ ,  $n = 3$ , adeoq;  $A = P - Q - R - S$ . Sed cum quantitas  $2 m$  infinite parva ſeu potius nulla, in Denominatorē termini tertii per quem  $d^n B$  multiplicatur, extet, Quantitas deſignata per  $P$  infinita eſt; atque ob eandem cauſam, Quantitas deſignata per  $-S$  infinita evadit, adeoque Quantitates  $A$ ,  $-Q$ ,  $-R$  evanescunt: Igitur  $P = S$ ,

diviſaque æquatione per  $\frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2}$  fit

$$d^n B \text{ in } \frac{2m-3}{2m} = \frac{dd}{m} x^{m-3} y^3 \text{ ſeu } d^n B \text{ in } \frac{2m-3}{2}$$

$= dd x^{m-3} y^3$ : ſcriptisque 0 & 3 pro  $m$  &  $n$  prodiſit

$$d B \text{ in } - \frac{3}{2} = \frac{y^3}{x^3}, \text{ seu } B = - \frac{2y^3}{3x^3},$$

### Corollarium 3<sup>um.</sup>

Si  $m$  ponatur aequalis termino cuivis sequentis seriei, — 2, — 1, 0, 1 2, 3, 4, 5, &c. quadratura Curvæ cujus ordinata  $x^m \sqrt{dx-xx}$ , pendet a quadratura Circuli: Area vero Curvæ cujus ordinata  $x^m \sqrt{dx+xx}$  pendet a quadratura Hyperbolæ, & relatio istius Curvæ cum Circulo aut Hyperbola exhibetur per Seriem nostram in terminis finitis.

### Corollarium 4<sup>um.</sup>

Si  $m$  exponatur per alium quemvis terminum differentem ab iis quas supra memoravimus, Curva cujus ordinata  $x^m \sqrt{dx-xx}$  aut  $x^m \sqrt{dx+xx}$ , neque quadratur exakte, nec ab Hyperbola aut Circulo pendet, sed ad Curvam simpliciorem reducitur per seriem nostram.

### Theorema 2<sup>um.</sup>

Sit  $A$  Area Curvæ cujus Abscissa  $x$  & ordinatim applicata  $\frac{x^m}{\sqrt{dx-xx}}$  Sit  $B$  area Curvæ cujus Abscissa eadem cum priori sed ordinatim applicata  $\frac{x^{m-n}}{\sqrt{dx-xx}}$  ponatur  $\sqrt{dx-xx} = y$ . Erit  $A = d^n B$

( 1117 )

$$\cdot d^n B \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} \text{ in } \frac{2m-7}{2m-6} \text{ &c. } = P.$$

$$- \frac{1}{m} \frac{x^{m-1}}{x} \quad y = - Q.$$

$$- \frac{d}{m-1} \text{ in } \frac{2m-1}{2m} x^{m-2} \quad y = - R$$

$$- \frac{dd}{m-2} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} x^{m-3} \quad y = - S$$

$$- \frac{d^3}{m-3} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} x^{m-4} \quad y = - T$$

&c.

Observationes ad primum Theorema, hic & in sequentibus locum habent.

### Corollarium I<sup>um.</sup>

Si  $m$  ponatur æqualis Termino cuivis sequentis seriei,

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \text{ &c. quadratura Curvæ cujus ordinatim ap-$$

plicata }  $\frac{x^m}{\sqrt{d x - xx}}$  aut  $\frac{x^m}{\sqrt{d x + xx}}$  finita evadit, & exhibetur per hanc

seriem.

*Corollarium 2<sup>um</sup>.*

Si  $n$  ponatur æqualis Termino cuivis sequentis seriei 1, 2, 3, 4, 5, 6, 7, &c. Curva omnis cujus ordinatim applicata

$\frac{x^n}{\sqrt{dx-xx}}$  aut  $\frac{x^n}{\sqrt{dx+xx}}$  quadratur per hanc seriem in terminis finitis.

*Corollarium 3<sup>um</sup>.*

Si  $m$  exponatur per terminum quemlibet sequentis seriei, 0, 1, 2, 3, 4, 5, 6, 7, &c. Curva cujus ordinatim applica-

cata  $\frac{x^m}{\sqrt{dx-xx}}$  pendet a Quadratura Circuli. Curva vero cujus

ordinatim applicata  $\frac{x^m}{\sqrt{dx+xx}}$  a quadratura Hyperbolæ. Etenim

si Centro  $C$ , Diametro  $AB = d$  describatur Circulus  $AEB$ , ac sumatur  $AD = x$ ; erecto  $DE$  normaliter, juge  $CE$ . Sector  $AEC$  per  $\frac{1}{2}dd$  divisus æqualis est Areæ Curvæ cujus Ordinata

$\frac{x^0}{\sqrt{dx-xx}}$  Eodem modo, si Centro  $C$ , Transverso axi  $AB = d$ ,

describatur æquilatera Hyperbola  $A E$ , sumatur  $AD = x$ , erigatur  $DE$  ad angulos rectos, jungatur  $CE$ ; sector  $ACE$  per  $\frac{1}{2}dd$  divisus æqualis est Areæ Curvæ cujus ordinata

$\frac{x^0}{\sqrt{dx+xx}}$

*Corol-*

Corollarium 4<sup>um.</sup>

Si  $m$  ponatur æqualis Termino cuivis, qui non in limitationes

præcedentes cadat, Curva cujus ordinata  $\frac{x^m}{\sqrt{d x + x^2}}$

neque quadratur exæcte,, nec a Circulo aut Hyperbola pendet,  
sed ad Curvam simpliciorem reducitur.

Theorema 3<sup>um.</sup>

Sit  $A$  Area Curvæ cujus Abscissa  $x$ , ordinatim applicata  $x^m \sqrt{r r \cdot x x}$ ,  
sit  $B$  area Curvæ cujus Abscissa itidem  $x$ , ordinatim applicata  
 $x^{m-2} \sqrt{r r \cdot x x}$ , ponatur  $\sqrt{r r \cdot x x} = y$ . Erit.  $A =$

$$r^2 n B \text{ in } \frac{m-1}{m+2} \text{ in } \frac{m-3}{m} \text{ in } \frac{m-5}{m-2} \text{ in } \frac{m-7}{m-4} \text{ &c. } = P.$$

$$- \frac{1}{m+2} x^{m-1} y^3 = - Q$$

$$- \frac{r r}{m} \text{ in } \frac{m-1}{m+2} x^{m-3} y^3 = - R$$

$$- \frac{r^4}{m-2} \text{ in } \frac{m-2}{m+2} \text{ in } \frac{m-3}{m} x^{m-5} y^3 = - S.$$

&c.

Corollarium 1<sup>um.</sup>

Si  $m$  exponatur per terminum quemvis sequentis seriei 1, 3,  
5, 7 9, &c. Quadratura Curvæ cujus ordinata  $x^m \sqrt{r r \cdot x x}$  aut  
 $x^m \sqrt{r r + x x}$  finita evadit, & exhibetur per hoc Theorema.

Corol-

**Corollarium 2<sup>um.</sup>**

Si  $n$  exponatur per terminum quemvis sequentis seriei 2, 3, 4, 5, 6, &c. Curva cujus ordinata  $x^{-2n} \sqrt{rr-xx}$  aut  $x^{-2n} \sqrt{rr+xx}$ , quadratur exakte per hoc Theorema.

**Corollarium 3<sup>um.</sup>**

Si  $m$  exponatur per Terminum quemvis sequentis seriei — 2, 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinata  $x^m \sqrt{rr-xx}$ , pendet a Circulo. Quadratura vero Curvæ cujus ordinata  $x^m \sqrt{rr+xx}$ , pendet ab Hyperbola.

**Corollarium 4<sup>um.</sup>**

Si  $m$  exponatur per Terminum quemvis differentem ab illis quos supra memoravimus, Curva cujus ordinata  $x^m \sqrt{rr-xx}$ , aut  $x^m \sqrt{rr+xx}$ , neque exakte quadratur, nec a Circulo aut Hyperbola pendet, sed ad simpliciorem Curvam reducitur.

**Theorema 4<sup>um.</sup>**

Sit  $A$  Area Curvæ cujus abscissa  $x$ , ordinatim applicata  $\frac{x^m}{\sqrt{rr-xx}}$ , Sit  $B$  Area Curvæ cujus Abscissa isti iam  $x$ , Ordinatim applicata  $\frac{x^{m-2n}}{\sqrt{rr-xx}}$  Erit  $A =$

$$r^{\frac{m-1}{m}} B \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} \text{ in } \frac{m-7}{m-6} \text{ &c. } = P.$$

$$- \frac{x^{\frac{m-1}{m}}}{m} y = - Q$$

$$- \frac{rr}{m-2} \text{ in } \frac{m-1}{m} x^{\frac{m-3}{m}} y = - R$$

$$- \frac{r^4}{m-4} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} x^{\frac{m-5}{m-2}} y = - S$$

$$- \frac{r^6}{m-6} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} x^{\frac{m-7}{m-4}} y = - T.$$

&c.

### Corollarium I<sup>um</sup>.

Si  $m$  exponatur per terminum quemvis sequentis seriei 1, 3, 5, 7, 9, &c. Quadratura Curvæ cujus ordinata

$\frac{x^m}{\sqrt{rr+xx}}$  aut  $\frac{x^m}{\sqrt{rr+xx}}$ , per hoc Theorema habetur in finitis

Terminis

### Corollarium 2<sup>um</sup>.

Si  $n$  exponatur per terminum quemlibet sequentis seriei 1, 2, 3, 4, 5, 6, &c. Curva cujus ordinatam applicata

$\frac{x^{-2n}}{\sqrt{rr+xx}}$  aut  $\frac{x^{-2n}}{\sqrt{rr+xx}}$  exacte quadratur per hoc Theorema

$\sqrt{\frac{rr+xx}{rr-xx}}$

M m m m m m m

Co

**Corollarium 3<sup>um.</sup>**

Si  $m$  exponatur per terminum quemvis sequentis seriei 0, 2, 4, 6, 8, 10, &c. Quadratura Curvæ, cuius ordinatim appli-

cata  $\frac{x}{\sqrt{rr-xx}}$  pendet a quadratura Circuli. Etenim si Centro C ra-

dio  $C A = r$  describatur Circulus  $A E G$ , sumatur  $C D = x$ , erigatur  $D E$  normalis ad  $C D$ , Jungatur  $C E$ : Sector  $C A E$  per  $\frac{1}{rr}$  divisus æqualis est Areæ Curvæ cuius ordinatin. ap-

plicata  $\frac{x^0}{\sqrt{rr-xx}}$ . Eodem modo si Centro C, Transverso famiaxi

$C A = r$ , describatur æqualatera Hyperbola  $E A M$ , ducatur  $C F$  ad  $A C$  perpendicularis  $= x$ , ducatur  $F E$  axi parallelia donec occurrat Hyperbolæ in  $E$ , jungatur  $C E$ : sector Hyperbolicus  $A C E$  per  $\frac{1}{rr}$  divisus æqualis est Areæ Curvæ cuius ordi-

natim applicata  $\frac{x^q}{\sqrt{rr+xx}}$

**Corollarium 4<sup>um.</sup>**

Si  $m$  exponatur per terminum quemlibet a præcedentibus

differentem, Curva cuius ordinata  $\frac{x^m}{\sqrt{rr-xx}}$  aut  $\frac{x^m}{\sqrt{rr+xx}}$  neque quadratur exakte, nec a Circulo aut Hyperbola pendet, sed ad Curvam simpliciorem reducitur,

Theorema 5<sup>um.</sup>

Sit  $A$  Area Curvæ cujus abscissa  $x$ , ordinatim applicata  $\frac{x^m}{d-x}$ ; sit  $B$  Area Curvæ cujus abscissa itidem  $x$ , ejusq; ordinatim

applicata  $\frac{x^{m-n}}{d-x}$  Erit Area

$$A = d^n B - \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} - \frac{ddx^{m-2}}{m-2} \text{ &c.}$$

Sit ordinatim applicata  $\frac{x^m}{d+x}$ , tunc Area erit =

$$A = \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} + \frac{ddx^{m-2}}{m-2} \text{ &c.} \pm d^n B.$$

## Corollarium.

Si  $m$  exponatur per terminum quemlibet sequentis seriei, o, 1, 2, 3, 4, 5, 6, &c. Quadratura Curvæ cujus ordinatim applicata  $\frac{x^m}{d-x}$ , aut  $\frac{x^m}{d+x}$  pender a quadratura Hyperbolæ;

Vide Fig. 3.

Etenim duos  $D E$ ,  $E F$  ad angulos rectos, sumatur  $EG = d$ , ducatur  $GH$  normalis ad  $EF$  & ipsi æqualis. Intra Asymptotos  $DE$ ,  $EF$  describatur Hyperbola per  $H$  transiens, quo facto sumatur  $HK = x$  versus  $E$  pro primo casu, at versus  $F$  pro secundo; ducatur ordinatim applicata  $KL$ : Area  $H G K L$  per  $dd$  divisa æqualis est Areæ Curvæ cujus ordinatim applicata

M m m m m m z ta

$\frac{x^0}{d-x}$  aut  $\frac{x^0}{d+x}$ . Hinc Solidum generatum a portione Ciffo-

dis dum circa Diametrum circuli genitoris revolvit, in finitis terminis exhibetur, data Hyperbolæ Quadratura.

### Theorema 6<sup>um.</sup>

Sit **A** Area Curvæ cujus abscissa  $x$ , ordinatim applicata

$\frac{x^m}{rr+xx}$ ; Sit **B** Area Curvæ cujus abscissa itidem  $x$ , ordinatim

applicata  $\frac{x^{m-2n}}{rr+xx}$ , Erit Area

$$A = \frac{x^{m-1}}{m-1} - \frac{rr x^{m-3}}{m-3} + \frac{r^4 x^{m-5}}{m-5} \text{ &c. } \mp r^{2n} B.$$

### Corollarium

Si  $m$  exponatur per terminum quilibet sequentis seriei  $0, 2, 4, 6, 8, \text{ &c.}$  Quadratura Curvæ cujus ordinatim ap-

plieata  $\frac{x^m}{rr+xx}$  pendet a rectificatione circularis Arcus. Etenim si

centro  $C$  radio  $CA = r$  describatur Circulus  $AEG$ , ducatur Tangens  $AK = x$  jangatur  $C K$  peripheriæ occurrentis in  $E$ ; arcus  $AE$  per  $rr$  divitus æqualis est Areæ curvæ cujus ordinata

$$\frac{x^0}{rr+xx}$$

Corol-

*Corollarium generale ad hæc sex Theorematæ.*

Curva omnis mechanica cuius quadratura rendet ab aliqua  $\Theta$  Curvis numero infinitis, cuius ordinatæ formæ lequentes ad ipsi

$$\text{possunt } x^m \sqrt{dx \pm xx}, \frac{x^m}{\sqrt{dx \pm xx}}; x^m \sqrt{rr \pm xx} \frac{x^m}{\sqrt{rr \pm xx}}$$

$\frac{x^m}{d \pm x}, \frac{x^m}{rr \pm xx}$ , per series has quadrari potest. Hoc Exemplo

unico indicare satis erit.

Posito quod Cubus Arcus Circularis Sinui verso correspondentis fiat Ordinata Curvæ, cuius Abscissa s. idem Sinus versus. Inquirenda est Arca istius Curvæ.

Sit Abscissa  $x$ , arcus circularis  $v$ , fluxio Areæ fit  $v^3 \dot{x}$ ;

Sit Area  $v^3 x - q$ . Igitur  $v^3 \dot{x} + 3v^2 \dot{x} \dot{x} - q = v^3 \dot{x}$ ,

$$\text{unde } q = 3v^2 \dot{x}; \text{ sed } \dot{x} = \frac{d \dot{x}}{2 \sqrt{dx - xx}}, \text{ igitur } q = \frac{3d v^2 x \dot{x}}{2 \sqrt{dx - xx}},$$

$$\text{sed per Theorema II. } \frac{x \dot{x}}{\sqrt{dx - xx}} = \frac{d \dot{x}}{2 \sqrt{dx - xx}} - \dot{x} = \dot{x} - \dot{x},$$

$$\text{adeoq; } \dot{x} = \frac{1}{2} d v^2 \dot{v} - \frac{1}{2} \dot{d} v^2 \dot{v}, \text{ igitur } q = \frac{1}{2} d v^3 - \int \frac{1}{2} d v^2 \dot{v},$$

Ergo ad hoc perventum est ut fluentem quantitatem inventamus cuius fluxio est  $\frac{1}{2} d v^2$ ,

Sit hæc quantitas  $\frac{1}{2} d v^2 y - r$ .

$$\text{Igitur } \frac{1}{2} d v^2 \dot{y} + \frac{1}{2} d v^2 y \dot{v} - \dot{r} = \frac{1}{2} d v^2 \dot{v},$$

$$\text{Adeoque } \dot{y} = \frac{1}{2} d v^2 y \dot{v} = \frac{1}{2} d d v \dot{v}. \text{ Sit } r = \frac{1}{2} d d v x - s.$$

$$\text{Igitur } \frac{1}{2} d d v \dot{x} = \frac{1}{2} d d v \dot{v} + \frac{1}{2} d d x \dot{v} - \dot{s}.$$

$$\text{adeoque. } \dot{v} = \frac{1}{2} d d x \dot{v} = \frac{3 d^3 x \dot{x}}{4 \sqrt{dx - xx}} = \frac{1}{4} d^3 v - \frac{3}{4} d^3 \dot{v},$$

per 2<sup>um</sup> Theorema.

Igi-

Igitur  $s = \frac{1}{4} d^3 v - \frac{1}{4} d^3 y$ . adeoque area quaesita =  $v^3 x - \frac{1}{2} d v^3 + \frac{1}{2} d v^2 y - \frac{1}{2} d d v x + \frac{1}{2} d^3 v - \frac{1}{4} d^3 y$ .

Quoniam autem Solida ex rotatione Curvarum genita, Superficies ab eadem rotatione genitae, Longitudines Curvarum, & Centra Gravitatis horum omnium a Quadratura Curvarum pendent, hæc si a Curvis supradictis pendent facillime computantur.

Postquam Theorematæ hæc concinnaveram, eaque Clarissimo *Newtono*, ut supremo harum rerum Judici, monstraveram; obtulit ille mihi Chartas suas manuscriptas, quibus mihi constat se diu compotem fuisse methodi qua, æquatione Trinomiali quavis data naturam Curvæ exprimente, illa Curva aut quadratur aut ad simpliciorem Curvam reducitur.

Op' andum autem esset ut non solum ea quæ ad hanc rem spectant, sed alia multa præclara ejus inventa publici juris facere dignaretur. Hoc credo universe Reipublicæ Literariæ votum esse.

Nullus dubito Doctissimos viros quorum scripta in actis eruditorum alibique tam valde Mathematicas disciplinas promoverunt, methodos huic nostræ affines habere; adeoque nihil in his mihi ascribendum puto nisi quod Theorematæ hæc reperierim, nescius an ullibi extarent; eaque ad formam tam facilem reduxerim, ut calculus omnis ad hanc materiam spectans uno quasi intuitu conficiatur. Prisquam scribendi finem facio, non abs re futurum esse arbitror, si nunc, nulla data citius occasione, pauca quædam reposuerim Clarissimi *Leibnitii* animadversionibus, ad Seriem quandam a me publicatam de radice infinitæ æquationis invenienda. Existimat Vir Clar. Seriem illam non satis generalem esse, utpote non attingentem casus ubi quantitates  $z$  &  $y$  in se invicem ducuntur; adeoque seriem aliam pro mea substituit, hancq; afferit mea infinite generaliorem: illum autem in levem hunc errorem inductum esse suspicor, quod quantitates  $a$ ,  $b$ ,  $c$ ,  $d$ , &c. pro quantitatibus datis assumperit, cum pro quantitatibus datis aut indeterminatis indiscriminatim usurpandæ fuerint. Sed exemplum unum afferre libet, quo pateat seriem nostram casus omnes pervadere; sit  $\text{Æquatio } nyz - z^3 = y^3$ . In Theoremate nostro fiat  $a = ny$ ,  $b = o$ ,  $c = -1$ ,  $g = o$ ,  $b = o$ ,  $i = 1$ , aut melius fiat  $g = yy$

$g = yy$ ,  $b = 0$ ,  $i = 0$ . in utroque casu fiet  $Z =$

$$Z = \frac{yy}{n} + \frac{y^3}{n^4} + \frac{3y^5}{n^7} + \frac{12y^7}{n^{10}} \text{ &c.}$$


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## V.

### *An Account of the Appearance of several Unusual Parhelia, or Mock-Suns, together with several Circular Arches lately seen in the Air by E: Halley.*

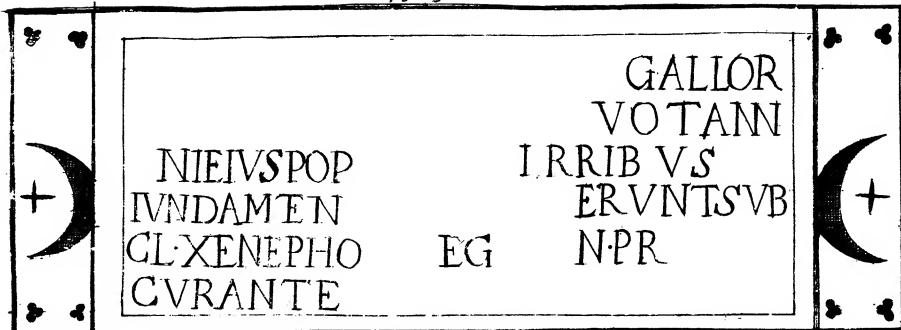
**O**N the Eighth of April, this present Year, 1702, walking in London Streets about ten in the Morning, the Air being clear, I observed the Sun to shine faintly, or as we call it, waterish; whereupon casting up my Eye, I perceived several Arches of Circles about him. I made what hast I could to get on the top of a House, which I did at Mr. Mordens by the Royal Exchange, and found the Appearance as is described in Figure 4. Tab. 3° wherein

$S$  is the true Sun,  $Z$  the Zenith.

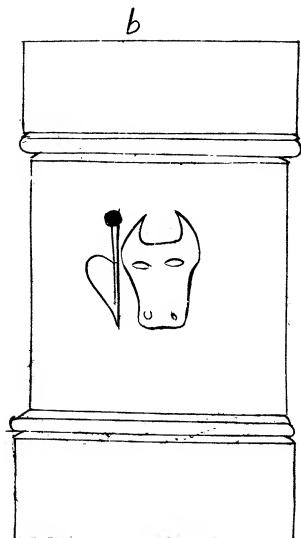
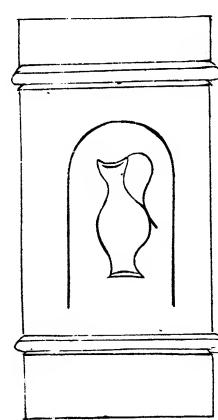
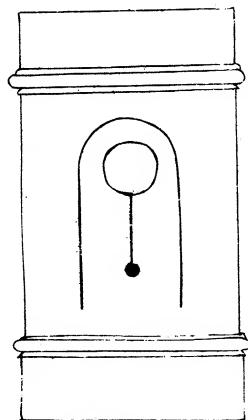
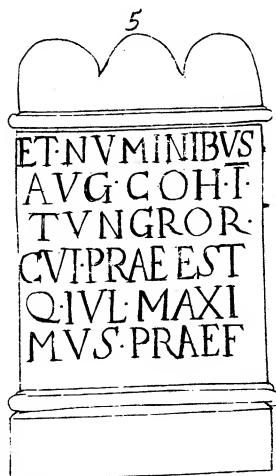
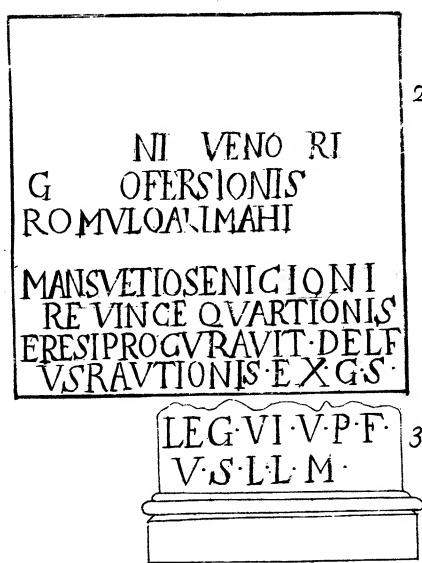
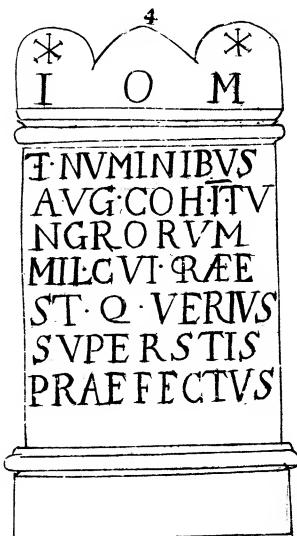
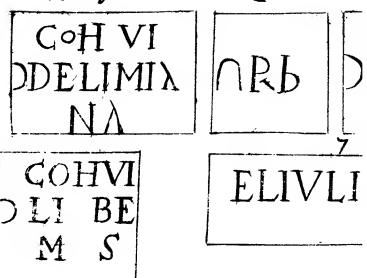
$S\, T\, P\, P$  a great white Circle passing through the Sun, and as near as I could judge, parallel to the Horizon. It was very distinct and entire, about two Degrees broad in the Northern part about  $T$ ; and held much the same breadth in the East and West, but grew narrower towards the Sun, its edges were not very well defined, the whole appearing like a faint white Cloud, and a part of it would have been taken for such, but the whole Circle seen in the pure Azure Sky was a very surprizing sight.

$V\, N\, X\, T$  a *Halo*, or rather *Iris*, that was likewise an intire Circle, having the Sun for its Center. I measured the Semidi- ameter of this to be much about 22 Degrees: the breath of this Arch which was well defined, was by estimate equal to the Suns Diameter, and it was coloured with the Colours of the *Iris*, but nothing near so *vivid* as in the common Rainbow. The *Reds* were next the Sun, and the *Blews* in the outward Limb. Within this Circle the Sky appeared somewhat obscure, especially near the Arch; and I take it, that the cause of that

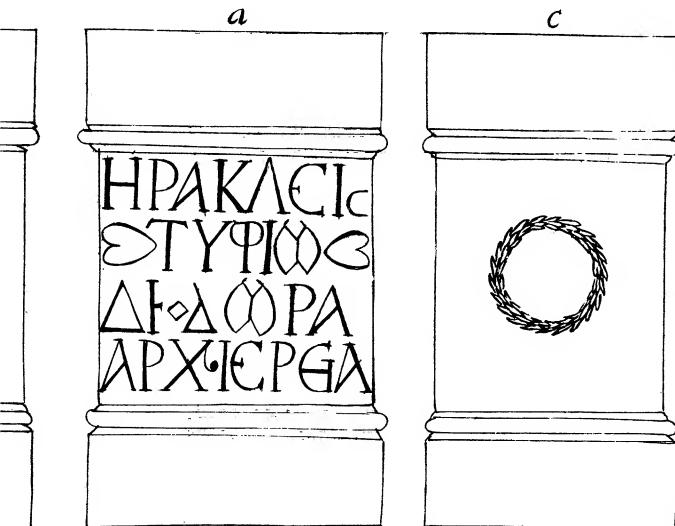
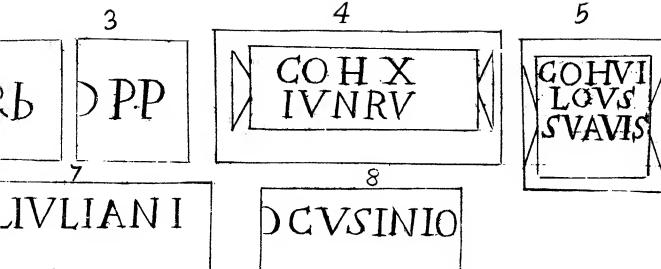
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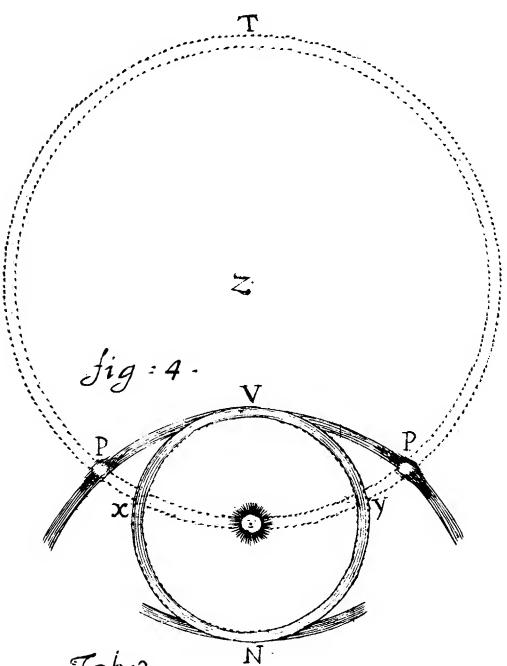
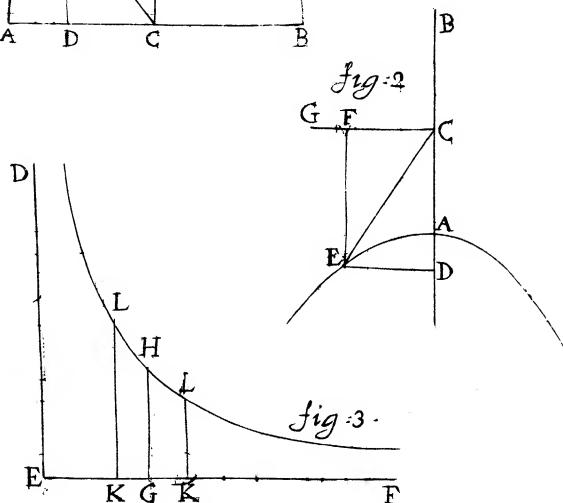
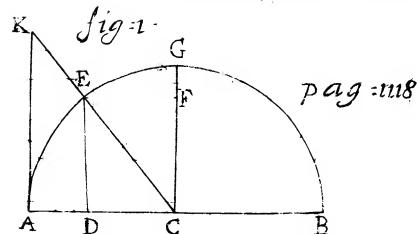


Tab: 1.



Tab :2

Philos. Transact. N° 278



Tab:3 -

